Worksheet 8

Name: ____________________________ Score: __________

1 Old Stuff (before §6.2)

1. The Table!!!

Linear functions	$m \times n$ Matrix	n vectors in $\mathbb{R}^m$
$f : \mathbb{R}^n \to \mathbb{R}^m$	$[f(e_1) \cdots f(e_n)]$	$\{f(e_1), \ldots, f(e_n)\}$
one to one	No free var's in RREF	linearly indep
onto	No zero rows in RREF	$Span(v_1, \ldots, v_n) = \mathbb{R}^m$
isomorphism	RREF is identity!	Basis for $\mathbb{R}^m$

2. Solve the system of linear equations

- $$\begin{bmatrix} 0 & 4 & 4 & -24 \\ -3 & -7 & -18 & 13 \\ -3 & -2 & -14 & -21 \end{bmatrix}$$

  [1, 0, 0, -5], [0, 1, 0, -10], [0, 0, 1, 4]

- $$\begin{bmatrix} 0 & 1 & 3 & 3 \\ 1 & -3 & -12 & 9 \\ 0 & 1 & 4 & -2 \end{bmatrix}$$

  [1, 0, 0, 3], [0, 1, 0, 18], [0, 0, 1, -5]

- $$\begin{bmatrix} 1 & -4 & -10 & 5 \\ 0 & 5 & 15 & -15 \\ -4 & 0 & -9 & 34 \end{bmatrix}$$

[1, 0, 0, 5], [0, 1, 0, 15], [0, 0, 1, -6]

- $$\begin{bmatrix} 1 & 0 & 1 & 5 \\ -3 & 1 & 2 & -21 \\ -4 & 0 & -3 & -22 \end{bmatrix}$$

[1, 0, 0, 7], [0, 1, 0, 4], [0, 0, 1, -2]

- $$\begin{bmatrix} -4 & 1 & -1 & -5 \\ -16 & 4 & -4 & -20 \\ -19 & 5 & -3 & -23 \end{bmatrix}$$

[1, 0, 2, 2], [0, 1, 7, 3], [0, 0, 0, 0]

3. Multiply these matrices.

- $$\begin{bmatrix} -1 & 13 & 5 \\ 0 & 12 & 11 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ -1 & 3 & -5 \\ 9 & 8 & 9 \end{bmatrix}$$

- $$\begin{bmatrix} -1 & 1 & 13 & -4 \end{bmatrix} \begin{bmatrix} 12 & 11 \\ -11 & 1 \\ 9 & -11 \\ 1 & 111 \end{bmatrix}$$

- $$\begin{bmatrix} -2 & 23 & 2 \\ -7 & 7 & 14 \\ 15 & 6 & -9 \end{bmatrix} \begin{bmatrix} 15 & 13 \\ 12 & -8 \\ 8 & 6 \end{bmatrix}$$

4. Find basis for $\operatorname{Col} A$ and $\operatorname{Nul} A$

- $$\begin{bmatrix} -1 & 2 & 5 \\ -1 & 1 & 2 \\ 4 & -4 & -7 \end{bmatrix}$$

[1, 0, 0, 0], [0, 1, 0, 0], [0, 0, 1, 0]

- $$\begin{bmatrix} 2 & 3 & 0 \\ 2 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$$

invertible!

- $$\begin{bmatrix} 4 & 4 & 1 \\ 0 & 1 & 4 \\ 3 & 3 & 1 \end{bmatrix}$$

  invertible!

- $$\begin{bmatrix} 1 & 1 & 0 \\ 2 & 3 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

  kills [1, -1, 1]

5. Is the last vector $\vec{b}$ in the span of the first three vectors $\vec{u}_1, \vec{u}_2, \vec{u}_3$?

- $$\begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -7 \end{bmatrix}, \begin{bmatrix} 6 \\ 1 \\ -17 \end{bmatrix}, \begin{bmatrix} 12 \\ 0 \\ -48 \end{bmatrix}$$

  No

- $$\begin{bmatrix} 12 \\ 9 \\ 0 \end{bmatrix} \begin{bmatrix} 9 \\ 7 \\ 0 \end{bmatrix} \begin{bmatrix} 30 \\ 23 \\ 0 \end{bmatrix} \begin{bmatrix} 36 \\ -5 \\ 7 \end{bmatrix}$$

  No

- $$\begin{bmatrix} 6 \\ 1 \\ 6 \end{bmatrix} \begin{bmatrix} 6 \\ 0 \\ 6 \end{bmatrix} \begin{bmatrix} 21 \\ 1 \\ 20 \end{bmatrix} \begin{bmatrix} -3 \\ 1 \\ -4 \end{bmatrix}$$

  yes

- $$\begin{bmatrix} 9 \\ 1 \\ 9 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \\ 8 \end{bmatrix} \begin{bmatrix} 18 \\ 3 \\ 22 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ -10 \end{bmatrix}$$

  yes

6. Find the inverse of this matrix. (Recall the 2x2 trick that saves time!) Find their determinants as well.

- $$\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

- $$\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$$
- $$\begin{bmatrix} 3 & 1 \\ 4 & 1 \end{bmatrix}$$
- $$\begin{bmatrix} 1 & 3 & 3 \\ 0 & 1 & 2 \\ 1 & 2 & 0 \end{bmatrix}$$
- $$\begin{bmatrix} 1 & 4 & 2 \\ 2 & 1 & 2 \\ 0 & 4 & 1 \end{bmatrix}$$
- $$\begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \\ 4 & 0 & 3 \end{bmatrix}$$

7.

8. Find the determinant. Recall the 2x2 and 3x3 formulas. Use it to say whether the matrix is invertible.

- $$\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$$
- $$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 4 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$
- $$\begin{bmatrix} 3 & 3 & 2 \\ 0 & 1 & 1 \\ 4 & 4 & 3 \end{bmatrix}$$
- $$\begin{bmatrix} 4 & 3 & 0 \\ 2 & 1 & 1 \\ 3 & 2 & 0 \end{bmatrix}$$
- $$\begin{bmatrix} 2 & 0 & 1 \\ 3 & 1 & 1 \\ 4 & 1 & 1 \end{bmatrix}$$

- $$\begin{bmatrix} 3 & 2 & 4 & 4 \\ 0 & 1 & 0 & 1 \\ 4 & 0 & 1 & 2 \\ 2 & 0 & 0 & 1 \end{bmatrix}$$
- $$\begin{bmatrix} 3 & 4 & 4 & 4 \\ 2 & 3 & 3 & 3 \\ 2 & 4 & 1 & 2 \\ 4 & 4 & 3 & 3 \end{bmatrix}$$

9. Find Eigenvectors and Eigenvalues:

- $$\begin{bmatrix} -16 & 36 \\ -6 & 14 \end{bmatrix}$$

  $$x_1' = -16x_1 + 36x_2$$
  $$x_2' = -6x_1 + 14x_2$$

  With P = [[2 3] [1 1]] And D = [[2 0] [0 -4]]
- $$\begin{bmatrix} -3 & 0 \\ -4 & 1 \end{bmatrix}$$

  With P = [[0 1] [1 1]] And D = [[1 0] [0 -3]]
- $$\begin{bmatrix} -4 & 1 \\ -2 & -1 \end{bmatrix}$$

  With P = [[1 1] [2 1]] And D = [[-2 0] [0 -3]]
- $$\begin{bmatrix} 15 & -12 & -2 \\ 19 & -16 & -2 \\ 6 & -6 & 2 \end{bmatrix}$$

  With P = [[2 2 1] [2 3 1] [1 1 0]] And D = [[2 0 0] [0 -4 0] [0 0 3]]

10. Find a basis of fundamental solutions to the differential equation associated to each matrix:

- $$\begin{bmatrix} 8 & -6 \\ 9 & -7 \end{bmatrix}$$

  With P = [[1 2] [1 3]] And D = [[2 0] [0 -1]]

- $$\begin{bmatrix} -13 & 12 \\ -9 & 8 \end{bmatrix}$$
  With P = [[1 4] [1 3]] And D = [[-1 0] [0 -4]]
- $$\begin{bmatrix} 2 & 0 \\ -4 & 3 \end{bmatrix}$$
  With P = [[1 0] [4 1]] And D = [[2 0] [0 3]]

11. Diagonalize the matrix:

- $$\begin{bmatrix} -3 & 5 \\ 0 & 2 \end{bmatrix}$$
  With P = [[1 1] [0 1]] And D = [[-3 0] [0 2]]
- $$\begin{bmatrix} 10 & -9 \\ 12 & -11 \end{bmatrix}$$
  With P = [[1 3] [1 4]] And D = [[1 0] [0 -2]]
- $$\begin{bmatrix} 2 & 0 & 0 \\ -13 & -9 & 8 \\ -15 & -12 & 11 \end{bmatrix}$$
  With P = [[0 1 0] [1 1 2] [1 3 3]] And D = [[-1 0 0] [0 2 0] [0 0 3]]
- $$\begin{bmatrix} -2 & 0 & 0 \\ -4 & -14 & 10 \\ -7 & -15 & 11 \end{bmatrix}$$
  With P = [[1 0 0] [3 1 2] [4 1 3]] And D = [[-2 0 0] [0 -4 0] [0 0 1]]